### Mathematical aspects of multimodal transportation

The goal of this chapter is to give a concise introduction into some mathematical aspects in the context of multimodal transportation.

The basic mathematical background is assumed, as well as the basic knowledge in system modelling, probability theory, mathematical statistics and information technology.

The chapter is divided into six sections which can be read consecutively.

#### Modes of transportation

By the general definition of transportation from Merriam-Webster the *transportation* is

* 1: an act, process, or instance of transporting or being transported
* 2a: means of conveyance or travel from one place to another
* b: public conveyance of passengers or goods especially as a commercial enterprise

There are also other interpretations of the term, but for the purposes of this chapter we accept this general definition.

The conventional European transportation modes are mentioned at the *Eurostats* site. Freight transport in the European Union (EU), covering the transport modes, is road, rail, air, maritime and inland waterways (Freight transport statistics - modal split, 2020).

The APICS Dictionary (Paul & Atwater, 2016) provides the following insight into multimodality: multimodal solutions ‒ transportation plans that involve multiple means of transportation and coordinate the physical and information requirements.

The best known definition of multimodal transportation is adopted from the United Nations Convention on International Multimodal Transport of Goods published in 1980. “International multimodal transport” means the carriage of goods by at least two different modes of transport on the basis of a multimodal transport contract from a place in one country at which the goods are taken in charge by the multimodal transport operator to a place designated for delivery situated in a different country”. The multimodal transport nowadays is not necessary is only international, so the definition is acceptable for the purpose of this chapter.

The transportation process is under consideration in frame of transportation systems of different scales – global, state, region or others. It is even more precise to discuss the network of transportation systems, as soon as it is practically not possible to distinguish a real independent transportation system.

The unique transportation case of freights or passengers may be provided in combined or multimodal mode. In multimodal transportation process various means of transportation are involved. The passenger planning the tour to Stockholm from Riga is actually planning the combined transportation, e.g. by train to Riga Central Station, by tram or taxi to Riga Passenger Terminal, by the ferry to Stockholm Passenger Terminal, and by bus or taxi to the hotel in the city centre. One trip from home to the hotel of destination is implemented as combination of several transportation modes.

There is a typical mistake of using the term *intermodal* instead of *multimodal*. The question is – is this process multimodal or intermodal transportation? The answer is very simple. There is a difference in terms of service provider. If you are planning your tour individually and buying all the tickets by yourself, you may identify your transportation as intermodal. If you are enjoying the tour operator service – just fix the start and the destination points of the trip – you get a single voucher for your trip from the tour operator, mentioning where, when, how and by whom you are transported at particular meeting points, then this trip looks like multimodal. The same approach helps to distinguish the freight transportation mode – a single service provider for a particular transportation case is in case of multimodal transportation. If the customer is planning the changes in transportation modes and making the choice of route, service provider etc. – intermodal transportation is under consideration. The subject of this chapter is multimodal transportation related mathematical issues.

The problems in the area of analysis of transportation systems are of various nature. Modern analysis approaches suppose the implementation of the various types of mathematical models. The models represent processes, activities, demand, alternative analysis, traffic assignment, traffic flows. The traffic flow models may differ by level of detail. The particular entities under consideration are typical for micro models. The high level of abstraction and consideration of flows rather than particular entities is the feature of macro models. And finally the combination of detailed description of some process features with the generalised approach to flow simulation are represented in mezo models (Barceló, 2010). In frame of these models, set of variables describes the transportation systems, providing the information about system state evolution over time. The model-based analysis approach in transportation area is relevant for any type of transportation, not only for the multimodal transportation. Thus, models are mentioned also further throughout the chapter.

Multimodal freight transportation has obvious advantages:

* flexibility,
* any type of container,
* route selection based on the customer's wishes,
* the ability to redirect during delivery,
* delivery "from door to door",
* monitoring at any stage of transportation,
* no need to search for multiple carriers,
* the authority transfer in organization of a complex process to the carrier and others.

The main advantage of multimodal transportation is the ability to take the benefit from each transportation mode. This may be not only low cost, but also acceptable delivery time and reliability, etc. These and other reasons explain the fact that multimodal transportation is the most modern type of delivery used in international transportation.

There are no principle disadvantages in multimodal transportation. However, there is a higher risk of damage or loss of goods due to the use of various types of transport, as well as to extra loading and unloading operations. The solution is in the choice of the reliable carrier.

#### Mathematical notations in descriptive multimodal transportation models

Transportation process, as research object, can be described in terms of discrete events. In discrete-event approach only the points in time at which the state of the system changes are considered.

In systems, which are analysed as discrete-event systems (DES), the operation of a system is interpreted as a chronological sequence of events. The events are the instant changes of system state. The events occur at particular, irregular time. The general event types associated with the multimodal transportation process comprise transportation start, finish, transportation mode change and other type of events if necessary. The transportation process is associated with its entity – the transportation unit (TU). Every passenger, parcel or freight unit are considered as TU. In the paradigm of DES the following multimodal transportation process definition could be provided: **Multimodal transportation process is the chronological sequence of events, each event is the state change and is managed by TU or manages the TU.**

The objects of the transportation process determine the special features of a process and natural units of measurement. The transportation process may be a single unique process or regular repeated process. The regular repeated process may have both constant and variable parameters.

The transportation process may be interpreted as a material flow. Material flows are inherent components of diverse systems. Material flows ensure both the interaction of system elements and system links with the environment. Thereby most of complex systems may be considered and analysed as material flow handling systems. The previous years publications apply such terms as "flow system", „material flow system”, „material handling” or „material flow handling system”. They state that the guarantee of effective interaction between the flow and manufacturing or distribution systems is an important task. The volume of the costs, related to control of the material flows between the elements of production or distribution system, is evaluated from 13 to 30 percent of total production or distribution costs. The concept of the flow system is wider than the concept of material flow system. The material flow system concept can be applied to describe the transportation systems as well. With the account of above mentioned, we could formulate the alternative definition of transportation system: the technically controlled material flow system, created to move the objects of material flows. Almost all up-to-date simulation software provide tools and components for material flow analysis, description, incorporation into simulations, obtaining and visualization of appropriate simulation results (Robinson, 2014).

The specification of transportation process could be made in the form of informal description, table, formalism, graphic scheme, or algorithm. The type of the specification is determined by the purposes of the application. Transportation process specification as a table is elementary transportation flow model. Table 1 shows example of such specification.

This tabular model can be also named the protocol of transportation process events. The protocol is based on the approach proposed for material flow protocol (Schenk, Tolujew, & Reggelin, 2008). The table may be also designed in a more compact form, however for the purpose of this section it is provided in the detailed form.

Here *i* is an ordinal number of the considered TU*i*, $i=\overbar{1,M}$. In this example variable *M* is a maximum of considered TU*,* *j* is a number of the event type, $j=\overbar{1,K}$, where *E*1 is transportation start event, *EK* – transportation finish event, and other events may be associated with transportation mode change or other relevant events in the transportation process of the freight under consideration. *K*-2 is a number of all possible events in the transportation process. The event time (*tij*) is a variable, corresponding to the occurrence of the *j*-th type of event related to the *i*-th TU.

Table 1. The protocol of the transportation process events

|  |  |  |
| --- | --- | --- |
| The ordinal number of a Transportation Unit in a process (*i*) | The event type (*Ej*) | The event time (*tij*) |
| 1 | *E*1 | *t*11 |  |  |  |  |  |
| 1 | *E*2 |  | *t*21 |  |  |  |  |
| … | … |  |  | … |  |  |  |
| 1 | *Ej* |  |  |  | *tj*1 |  |  |
| … | … |  |  |  |  | … |  |
| 1 | *EK* |  |  |  |  |  | *tK*1 |
| 2 | *E*2 | *t*12 | *t*22 |  | *tj*2 |  | t*K*2 |
| ... |  |  |  |  |  |  |  |
| *i* | *ti* | *t*1*i* | *t*2*i* |  | *tji* |  | *tKi* |
| ... |  |  |  |  |  |  |  |
| *M* | *tM* | *t*1*M* | *t*2*M* |  | *tjM* |  | *tKM* |

The transportation process *P*, described in Table 1, is a structure

$P=\left〈T,E,F\right〉$, (1)

Where *T* is a set of particular events times,

 *E* is a discrete set of events types, and

 *F* is a discrete set of TU*i*, $i=\overbar{1,M}$.

The protocol can be augmented with the relevant information about the TU, such as weight, volume, transportation and storage conditions and other properties.

The protocol can be visualized as time diagram, as shown in Figure 1.

In Figure 1 the transportation process events are shown as vertical line segments of different height corresponding to particular TU. However it is possible to use the height notation to introduce the volume, weight or container type of the TU. The time moments are according to the Table 1. It is possible to see the irregular time periods between particular events related to a concrete TU. Figure 1 provides *Δt*1*i* intervals between events of the TU1 process. The data may be used to for further simulations and as simulation validation data.

Creating transportation process specifications, the best results can be achieved by combination of different types of description. A universal transportation process description method or type could hardly be defined. Certain types of description, such as formal and graphic, are more applicable in the area of mathematical modelling.

The transportation process protocol is not the only form of the formal description proposed by researchers. This approach corresponds with the Event Logs that are obtained from the observations. As soon event log is a record of events, the correspondence between the process protocols an event logs is very close (van der Aalst, 2011).



Figure 1. Transportation process events over time

This form of process description can be used as a conceptual model for the simulation of the process. The process mining is also based on the event log. Process mining uses event data to extract process‑related information. The process mining is introduced as a technique to provide means for process improvement in various application domains. “Process mining is an emerging discipline providing comprehensive sets of tools to provide fact-based insights and to support process improvements. This new discipline builds on process model-driven approaches and data mining” (van der Aalst, 2011). As the most challenging task of the process mining the automatic generation of process models, based on the event log, is highlighted. The event log form of process description can be used for input data preparation for mathematical modelling. For example, extracting time moments from log or protocol provide the data for finding *interarrival* time distribution between freights. These interarrival times or time intervals are calculated as

$τ\_{j}= t\_{j+1,1}-t\_{j,1}, j=\overbar{1,M}$.

The variable *τ* (probability) distribution is relevant to simulate incoming flow of freight objects.

The section sets out the general requirements to the formal description of multimodal transportation process, which should fit the specific requirements. The qualitative description develops the basis for the analysis and formalization of transportation process at a conceptualization stage of simulation model.

#### Objectives of multimodal transportation

The effectiveness of multimodal transport is that it uses the main advantages of each type of transport: costs, speed, accuracy and environmental impact.

Multimodal transport has many advantages that correspond with the objectives of multimodal transportation in comparison with conventional unimodal transportation or intermodal transportation. These general objectives are:

* Total costs reduction
* Delivery time reduction
* Origin and destination points all over the world
* Security

The objectives of multimodal transportation do not differ in principle from the goals of any other types of transportation. These objectives can be considered from the point of view of optimizing the transportation process considering various goal functions.

#### Optimisation tasks in multimodal transportation

Since the goal of multimodal transportation of goods and passengers is to achieve the best possible results, it is quite logical that various optimization problems are considered in this area.

The optimisation in general is a mode of action, providing the choice from all possible options of resource usage to obtain the best results. This not exactly academic definition is emphasising the resource usage. The more specific mathematical formulation may be: the procedures or algorithms of finding the extreme value of a goal function involved in this choice. As soon as the goal function is mentioned, the choice procedure should be formalised.

There are also organisational or infrastructure-based ways of optimization of multimodal transportation such as (Деев & Корнилов, 2015):

* Application of a centralized system for the import and export of goods; development of an integrated network of transport, warehouses and logistic hubs; creation of multimodal logistics enterprises. As soon as these activities are infrastructure-based the investments are necessary.
* Creation of multimodal corridors and regional transport and logistics systems; expansion of the set of transport and forwarding services; modernization of warehouses and hubs; restructuring of transport enterprises. Both investments and infrastructure changes are assumed.
* Determination of rational areas of transport application or equal distances; calculation of the economic effect of the selected transport mode. Calculations become significantly more complicated. Formal definition of the transport route needs to be actualised.
* Freight flow forecasting. High possibility of forecast errors, as soon as the transportation parameters are dynamic.
* Bottleneck identification that is concerned with the analysis of infrastructure resource plans (Möller, 2014).

Bottleneck identification is concerned with optimization of intermodal and multimodal transportation chains, timely and concurrent use of resources, transaction analysis, multicriteria approach as well (Möller, 2014).

Coming back to the mathematical aspect, we assume that the optimisation tasks are solved with optimisation models. In frame of this section it is not realistic to provide the complete review of the types of optimisation models, however we introduce some basic concepts.

The first idea is about the optimisation models. Not only purely analytical models are used, other types of mathematical models are also suitable. The widely used mathematical optimisation models are algorithmic or simulation ones. In this case the optimisation algorithms give not “precise” optimal solution, as analytical tasks do. Simulation optimisation usually gives the approximate, or best possible solution. Simulation-based optimization is used in different types of transportation tasks. There is no unique recommended optimisation technique. The review (Zvirgzdiņa & Tolujevs, 2012) provides the summary of optimisation algorithms that are relevant to simulation optimisation of complex systems and is based on the Winter Simulation Conference publications over the long period of time.

Despite the fact that the applied optimisation tasks belong to completely different areas, they have a common form. The main stages of optimisation task formulation are the construction of the goal function, the definition of the set or the space of feasible solutions, and the set of constraints of control variables. The solution is obtained by changing the control variables, taking into account the constraints, and looking for the extreme value of the goal function.

The tasks are more complex in case of multi-criteria or multi-objective optimisation, when multiple goals are formulated. The typical fault formulation of the optimisation task is “to find the solution providing maximum profit by using minimum of resources”. The error is in the task formulation: to find the optimum of two contradicting values. The correct task formulation may be “to find the solution providing maximum profit by using a pre-defined amount of resources”. There is a possibility to formulate alternative task as well.

What are the specific features of optimisation tasks in multimodal transportation area? First of all it is necessary to consider the main performance indicators of the multimodal transportation process.

The performance of a process or organization can be defined in different ways. Typically, three dimensions of performance are identified: time, cost and quality (van der Aalst, 2011).

Other authors provide more detailed approach to the factors that may be taken into account to evaluate the performance of the transportation process (Udomwannakhet, et al., 2018). The factors that may be introduced into performance evaluation are: route length, environmental Impact, time window compliance and other factors. Most of these detailed factors can be expressed in terms of time, cost and quality.

For optimisation it is absolutely necessary that the goal function is measurable.

The formulation of a simple optimisation model of a multimodal transportation process is based on the process description and transportation goals.

Example

The example is based on the case described in (Гедрис, Оникийчук, & Гедрис, 2016). The transportation process under consideration is a process of one particular freight transportation from manufacturing facility to the regional distribution centre. There are various potential modes of transportation, such as transportation in a container by road, using (a) standard 40 feet container type or (b) 20 feet container type, (c) 40 feet high cube. The (d) railway transportation using 40 feet container, (e) multimodal transportation by road and railway using 20 feet container, and (f) multimodal transportation by rail and ship using 20 feet container. The efficiency of the selected scenario is evaluated by the following particular criteria:

* Transportation costs and costs of additional operations with cargo (reloading, insurance, customs), *TC*, currency units.
* Pipeline inventory costs, *PC*, currency units
* Risk of sudden transportation disruption costs (calculated based on the history of the transportation or expert evaluation), *DC*, currency units
* Transportation time or lead time, *L* , days

The current problem has a countable number of alternatives or alternative scenarios, so we have a discrete problem. The alternatives are formulated explicitly. The goal of the scenario evaluation is to minimize the values of all the partial criteria. The Table 2 shows the calculated values of individual criteria of the scenarios.

Table 2. The partial criteria values for scenarios

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Scenario | *TC* | *PC* | *DC* | *L* |
| a | 19974 | 9.24 | 1.5 | 2.2 |
| b | 21158 | 10.5 | 1.5 | 1.8 |
| c | 20293 | 8.4 | 1.4 | 2.1 |
| d | 17541 | 42 | 0.8 | 3.7 |
| e | 20977 | 85 | 1.0 | 4.1 |
| f | 19147 | 84 | 1.1 | 2.7 |

The first action that can be done after the simplified analysis of alternatives is the decision about not using scenario (e), as soon as it has all partial criteria values worse than scenario (f).

Analysis of total costs shows that the scenario (d) has the lowest total costs

*Cd*=*TC* + *PC* + *DC* = 17583.8 = min (*Ca*, *Cb*, *Cc*, *Cd*, *Cf*).

Thus, the goal function, including only costs, has minimum value for this scenario.

But in this case we have more than one criteria, and the second one is the transportation time and the functions are providing the common 2-dimensional vector criterion

*F*=(*C*,*L*).

The criterion *F* has values in the space of 2-dimensional vectors. All existing valid alternatives (a, b, c, d and f) can be described in terms of possible decision vector, for example alternative a has the criteria vector *F*a=(*C*a, *L*a).

In our case, the choice in the set of alternatives is mathematically equivalent to the choice in the set of vectors. All the definitions and results can be formulated both in terms of alternatives and in terms of vectors. It is always possible to make the transition from one form of presentation to another. A problem formulation includes a set of valid alternatives (a, b, c, d and f) and a vector criterion *F*, and now our example problem is formulated as a multi-criteria optimization problem.

The feasible alternative may be chosen based on Pareto optimality principle. Pareto optimal vector for multi-criteria alternative is one that has the higher (or lower) values of at least one of criteria in comparison with other vectors. Looking back at the example problem, we can see that there is the constraint or constraints missing for the second criterion – transportation time *L*. The scenario d has the best cost value *C*d, but the transportation time is the longest of all feasible solutions. If there are no constraints for this criterion, the alternative d is the solution. If the constraint is introduced for example

*L* ≤ 3,

then the Pareto optimal solution is the alternative f.

In conclusion of the example, we note that the introduction of additional choice criteria increases the dimension of the problem. However, high-dimensional problems can also be solved using the Pareto optimality principle, or by other methods. If it is possible to build simulation model of the process it is possible to use plethora of optimization methods. Algorithms, heuristics and iterative methods are applicable and the researchers have a vide choice.

In some cases multi-criteria methods are used that do not require mathematical models. The fundamental complexity of multi-criterial choice problems is that it is impossible to determine a priori what is called the best solution. If the number of alternatives is small, the selection is made using outranking methods (Лотов & Поспелова, 2014). It is also possible to use the preference for making the optimal choice. For advanced readers and deeper understanding of the issue the further reading is recommended, e.g. (Caramia & Dell’Olmo, 2008).

The modern trend in the transportation process quality control is the evaluation and control of environmental impact, as a part of overall business process quality management (An official website of the European Union, 2014). The most significant environmental impact associated with transportation is the production of greenhouse gases. The largest source of greenhouse gas emissions from human activities is from burning fossil fuels for electricity, heat, and transportation. Detailed emissions calculation on single freight or even parcel may be performed and included into the optimisation goal function. Most publications recommend to calculate the performance indicators for transportation process in particular, i.e. not taking into account other aspects of a business process.

The European Standard UNI EN 16258: 2013 defines a common methodology for the calculation and declaration of energy consumption and greenhouse gas (GHG) emissions related to any transport service (of freight, passengers or both). It is possible to provide detailed distance-based calculations of energy consumption and relevant greenhouse gas emissions for any transportation mode (Schmied & Knörr, 2012).

Multi-criteria is an integral feature of most real selection problems and requires special analysis methods. The decision-maker needs to study the Pareto principle application, which plays an important role in decision-making, as well as the theory of the relative importance of criteria, goal programming and analytic hierarchy process. The complexity of multimodal transportation processes suggests to decision makers in logistics to work with efficient solutions, mainly to capture different aspects. Thus, we recommend to study multimodal transportation processes with a specific focus on multi-criteria optimisation.

#### Mathematical models in multimodal transportation performance analysis

For the purposes of this chapter we assume the minimal acquaintance with system modelling and appropriate terminology. The term *model* is interpreted as any type of object replacing the research object. This replacement should be useful, i.e. the model provides the information about the object. The type of the model is related to the

* object properties (structure, layout, etc.),
* application area (teaching, training, etc.),
* formalisation level (descriptive, mathematical, graphical, simulation, etc.),
* implementation goal (control, forecast, identification, etc.),
* time factor (static, dynamic),
* randomness (deterministic, stochastic).

One can find more detailed approach to model classification as soon as there are lots of interpretations of model concept, types of models, modelling approaches and model application areas. This is true for multimodal transportation as well.

Deterministic models

The deterministic models suppose no random factors under consideration. Known parameter values, rules, thus the behaviour of the model is fully predictable under known initial conditions. Models of this type are used for all types of technical-engineering calculations.

Deterministic optimisation model of cost function is introduced in the previous section. The questions related to the evaluation of the costs are not covered in the example. We assume that the carriers are providing information on tariffs and transportation times, thus the costs can be calculated for each alternative. The optimisation model is fully deterministic.

However in relation of the multimodal transportation we can introduce other deterministic data-based models that are useful for alternative analysis.

To introduce formal problem statement for route assignment the road network topology is taken into account. The network description includes origin and destination nodes, intermediate nodes that are relevant to the specific transportation mode, nodes where transportation mode can be changed. The nodes are described with their coordinates, available connections to other nodes, and the length of these connections. It is possible to create a formal network description for the previous example using the graph theory.

We call a graph several nodes (vertexes), some pairs of them are linked by lines (edges). In frame of these chapter we consider non-empty graphs. The graph is called connected if each vertex is joined to any other with lines. A loop is a path along the edges of a graph that starts and ends at the same vertex. Also, a graph is called weighted if each edge corresponds to a number (weight). There can't be two edges connecting the same vertices (Diestel, 2017).

In this case only two paths have intermediate nodes, where transportation mode is changed. To use graph theory for the formal description of the transportation task we need to introduce



Figure 2. The graph of the example transportation process

intermediate nodes for alternatives a, b, and c. These nodes are necessary, as soon as in graph theory the nodes can be joined with a single line only. As soon as our process alternatives a, b, and c are unimodal, then the introduction of the fictive nodes supposes that the weight of the lines joining the origin and destination points should be assigned to two lines instead of a single line. So the graph used for the formal description is simple and the distances are determined in terms of transportation times. The nodes and links in the transportation network correspond with the vertices and edges of graph, and the distances, or travel times are interpreted as weights of the edges in graph theory. Weight is a numerical value, assigned to an edge of a graph.

The multimodal transportation alternative choice problem can be considered with a help of non-empty, final, directed, and weighted, graph *G* with the set of 8 vertices *V*, 16 edges *E* and a set of weights *W*:

*G* = (*V*, *E*),

*V* = {*O*, *I*a, *I*b, *I*c, *I*d, *I*e, *I*f, *D*},

*E* = {{*O*, *I*a}, {*I*a, *D*}, {*O*, *I*b}, {*I*b, *D*}, {*O*, *I*c}, {*I*c, *D*}, {*O*, *I*d}, {*I*d, *D*}, {*O*, *I*e}, {*I*e, *D*}, {*O*, *I*f}, {*I*f, *D*}},

*W* = {2.2, 0, 1.8, 0, 2.1, 0, 2.1, 1.6, 1.5, 2.6, 1.1, 1.6}.

The weights of graph edges correspond with the amount of effort needed to travel from one vertex to another. In our example the weights are travel times. In other applications the weights may be distances, fuel consumption, environmental impact and other labels.

In some logistics tasks the edges, vertices and weights are interpreted in a different way. The interpretation is based on the type of the problem to be solved.

There is a task that was not discussed within the example but it is supposed that the best or shortest rout of each alternative is considered. The shortest route finding problem solution is also can be found using a deterministic data-based model.

The best-known deterministic problem in transportation process analysis is a problem of route assignment. Knowing origin and destination points various routes are analysed. The choice of the feasible route is possible considering travel time, costs, and environmental impact. This type of models is not specific for multimodal transportation, routing tasks are relevant to all types of transportation.

Finding the shortest route is a vital task and is used almost everywhere, not only in transportation. The road transportation routes themselves are evaluated using map and road-based information. These routes may be evaluated by using the graph theory and relevant algorithms. The shortest route is searched between two specified nodes in the graph. We use the term “shortest” using distances between nodes, but we can formulate the best route finding in terms of fuel consumption, environmental impact, time or other factors of interest.

The most frequently used route optimisation algorithms are

* Dijkstra
* Floyd–Warshall
* Bellman–Ford
* And brute force search algorithm.

These algorithms are easily executed with a small number of nodes in the graph. As their number increases, the task of finding the shortest path becomes more complex.

The deterministic model can be transformed into stochastic one if deterministic parameters or factor values become random. In our example it is supposed that transportation times are known as constants. Nevertheless, it is obvious that transportation time subject to uncertainties. The same happens when we do not have perfect real-time information about cost, then the route choice decisions have uncertain factors.

Stochastic models

Most real life problems and tasks include variability. The variability or uncertainty may be of different nature. Some factors may be described as random variables. These factors are introduced into models by using probability functions. The values of such factors are generated by using probability functions and random numbers and are unpredictable. The methods and approaches for data collection, processing and generation are widely used for various types of stochastic simulations [Banks]. Weather conditions, traffic flow density, customer order arrivals are examples of unpredictable variabilities, that can be described and introduces into models by using probability functions or sampling from standard statistical distributions.

Some types of variabilities are not really unpredictable [Robinson]. The examples of these type of variabilities are scheduled events, such as times of switching the direction of traffic lanes. These type of variability may be introduced into the model with the help of schedule or event log.

The main types of stochastic models are simulation models and Monte-Carlo simulations.

The simulation models are mathematical, algorithmic and dynamic models created using the software. These models simulate the processes in real systems and may be created in frame of diverse approaches. The most known and advanced simulation approaches are considering the following types of systems:

* Discrete state and continuous time (or discrete-event)
* Discrete time and continuous state (system dynamics)
* Discrete rate (combining the discrete-event time advance technique with system dynamics flow intensities)
* Discrete time and state (cell automata)

There are lots of software products on the market that can satisfy the researchers’ needs in the area of transportation. However there are specific software features that can best fit some specific problem. The Institute for Operations Research and the Management Sciences provides a biannual software survey, accompanied by analytical article on the topic (Swain, 2019).

Discrete-event system simulation (DESS) provides the advanced possibilities to simulate processes at micro level of detail. The particular freights, vehicles, lanes and other resources are simulated as entities of interest. Models of this type can be created using data extracted from event logs. The processes of a real system are simulated based on the typical events of this system. The system process is a superposition of all entities processes during particular time period. Figure 3 provides the illustration of the system process.

It is clear from the illustration that even simple processes being put into a common system process are providing the comprehensive common process. The analysis of such a process is based on the concept of resource utilisation by entity and entity delays. Queues and vehicles also are interpreted as multiple-content resources. Modern DESS software provides various types of reports for analysis purposes.

However, it is important to understand, what the expected types of results provided by DESS are. As soon as process simulation is based on resource utilisation and delays the generic types of simulation results are:

* Resource and the whole system utilisation in terms of contents
* Resource and the whole system utilisation terms of time
* Entity time in resource, entity time in system.

As soon as simulation is mostly used for stochastic systems analysis, the models include variability, and these results are stochastic as well. So when we discuss resource utilisation we suppose that statistical data are obtained.

Resource utilisation in terms of contents shows the average number of entities in this resource. This value is calculated using information about how long the entity was there in the resource, or as weighted average. Usually maximum and minimum values are provided, frequency statistics may be available as well as time diagram of entities in the resource.



Figure 3. The superposition time diagram of particular entities processes in a system

Resource utilisation in terms of time is estimated as the ratio of time spent working to the time of observation. The average entity time in resource is the same as the average resource operation time. This time is evaluated as time spent working per all entities that used the resource during the time of observation, or as an arithmetic average. Usually utilisation is analysed in more detail: used, blocked or not working because of working schedule.

The typical simulation results time diagrams are shown in Figure 4. The time diagrams of the resource and queue contents of the process are shown. On all the diagrams the vertical axes is for content volume, and horizontal axis is for time value.

After discussion of the typical results considered in DESS, we can conclude that for multimodal transportation process simulation this simulation type is rather appropriate. The level of detail allows to introduce transportation mode change events into consideration, as well as to simulate relevant vehicles and container types, loading and unloading operations and various delays. The results of simulation can support decision about organisation of multimodal



Figure 4. Simulation results time diagrams of DESS software

process based on total transportation time, resource utilisation and extended with appropriate technical‑economical calculations.

If we return back to our simple example of transportation mode choice from 6 alternatives we can make it more realistic and useful if we introduce variability. The most realistic way to consider variability is to use statistical information about transportation time.

Example

For the example we use the alternative (f) of a previous example - multimodal transportation by rail and ship using 20 feet container. The alternative transportation time is considered as random variable *L*f and consists of three variables: transportation time by rail *t*r, transportation mode change time *t*c and transportation time by ship *t*s:

*L*f = *t*r + *t*c + *t*s

In previous example transportation mode change time *t*c is considered as a part of transportation time by ship *t*s. For the purpose of the current example it is considered as a particular time. These variable times are random, and we have only guesses about minimum and maximum values of these variables, for example

*t*r = 1.1 ± 0.11; *t*c = 0.4 ± 0.04; *t*s = 1.2 ± 0.12.

The cases where the real type of random variable is unknown are quite common. In such cases some assumptions about the variables are accepted. For this example the uniform distribution is assumed as a guess about the variable times *t*r , *t*c , and *t*s. The next step is the analysis of the random variable *L*f that can be performed with Monte Carlo sampling. As a result the statistical distribution of the random variable *L*f` is obtained and the risk of the late delivery may be evaluated. The Monte Carlo sampling results are shown in Figure 5.

 

Figure 5. The transportation time *L*f histogram from 100000 samples

The Monte Carlo sampling provides the way to evaluate the probability that the total transportation time is longer than expected. For the example the average *L*f value is 2.6997, while the probability evaluation of *L*f >=2.77 is 0.2490.

The assumptions about the random variables under consideration may be refined when additional information or more observations are obtained about these transportation times and their variability. However introduction of stochastic model of the transportation time provides additional utility to the model of transportation time.

The simple example demonstrates the achievement from introduction of stochastic aspect into consideration. The main advantage of the stochastic model is that it fully reflects the assumptions made. Stochastic models make it possible to make analytical conclusions in conditions when deterministic calculations are impossible or insufficient.

#### Multimodal transportation and the layer model

At the end of the twentieth Century Schoemaker (Schoemaker, Koolstra, & Bovy, 1990) proposed the framework for transportation model analysis. The basic model was formulated for passenger transportation. The basic model consists of three layers: Activities, Transport services and Traffic services. The layers are interconnected and may be interpreted as a transportation logistics system. Activities layer provides demand for transportation and receives the information about potential supply of demanded services. The intermediate layer − Transport service layer – receives the demand for transportation from Activities and information about potential supply from Traffic services. Transport service layer provides the demand for transportation aids for Traffic services and the information about potential supply of demanded services for Activities.

The transport services may be provided in various modes including multimodal transportation. The quality of the service depends on the available traffic services. The framework is mostly oriented on the public transport analysis. The multimodal transportation in the layer model is provided by the transport service integrator. The transport service itself is considered as the combination of service components and means of transport. The most important conclusion of the research is emphasising the role of transport service provider (integrator), as well as considering the activities and functions of transport service provider (Schoemaker, Koolstra, & Bovy, 1990).

The basic layer model is developed and updated by other researchers in more recent times (van Nes, 2002).

It should be mentioned that the term “layer model” in multimodal transportation has also other interpretations, e.g. the topology structure and the different layers of multimodal transport network introducing the layers of road, bike, foot, train and bus networks of the passenger multimodal transportation (Sossoe, 2017).

There is also more traditional layer models interpretation related to smart city architecture as layered architecture in transportation context (Deka & Mashrur, 2019).

#### The future of multimodal mobility

The trend of the development of multimodal transportation is the successful interaction between various transportation modes. The maximum effect of multimodal transportation can be obtained only by improving the overall performance of all stages of the transportation process, where all the stages are working as a single system. The new and smart services, the improvement of the existing and related services, the increased attention to environmental issues are organisational trends. The rapid evolution of the mathematical approaches in transportation organisation tends to make the multimodal transportation more available and preferable.

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